Reading 6: The Arbitrage Pricing Theory and Multifactor Models of Risk and Return

After completing this reading, you should be able to:

- Explain the arbitrage pricing theory (APT), describe its assumptions, and compare the APT to the CAPM.
- Describe the inputs (including factor betas) to a multifactor model and explain the challenges of using multifactor models in hedging.
- Calculate the expected return of an asset using a single-factor and a multifactor model.
- Explain how to construct a portfolio to hedge exposure to multiple factors.
- Describe and apply the Fama-French three-factor model in estimating asset returns.

In the previous reading, we discussed the Capital Asset Pricing Model (CAPM). CAPM is a single-factor model that gives the expected return of a portfolio as a linear function of the markets' risk premium above the risk-free rate, where beta is the gradient of the line.

On the other hand, the Arbitrage Pricing Model (APT) uses the same analogy as CAPM, but it includes multiple economic factors.

The Arbitrage Pricing Theory

According to APT, multiple factors (such as indices on stocks and bonds) can be used to explain the expected rate of return on a risky asset. APT has three common assumptions.

Assumptions of the APT model:

1. The returns from the assets can be explained using systemic factors.
2. No arbitrage opportunities exist in a well-diversified portfolio. (**Arbitrage** refers to the action of buying an asset in the cheaper market and simultaneously selling that asset in

the more expensive market to make a risk-free profit.)

3. By using diversification, the specific risks can be eliminated from the portfolios by the investors.

According to APT, return on given security *i* is given by:

$$R_i = E(Ri) + \beta_i 1 [I_1 - E(I_1)] + \cdots + \beta_i K [I_K - E(I_K)] + e_i$$

Where

R_i: rate of return on security i (i = 1, 2, …, N)

$E(R_i)$: the expected return of security i.

$I_K - E(I_K)$: Surprise factor (the difference between the observed and expected values in factor k)

β_{iK}: measure the effect of changes in a factor I_k on the rate of return of security i

e_i: noise factor also called idiosyncratic factor

The APT was put to trial by Roll and Ross (1980) and Chen, Roll, and Ross (1986) while determining the factors that explained the average returns on traded stocks on New York Securities Exchange (NYSE).

According to Roll, a well-diversified portfolio are volatile, and that the volatility of a long portfolio is equivalent to half of the average volatility of its constituent assets. Therefore, he concluded that systematic risk drivers limit the impact of diversification within the asset groups.

According to Ross (1976), assuming that there is no arbitrage opportunity, the expected return on a well-diversified is given by:

$$E(R_P) = E(R_Z) + \beta_P 1 [E(I_1) - E(R_Z)] + \cdots + \beta_{PK} [E(I_K) - (R_Z)]$$

where

$E(R_P)$: Expected return on a well-diversified portfolio

β_{PK}: Factor loading for portfolio relative of factor k

$E(R_Z)$: Expected rate of return on a portfolio with zero betas (such as risk-free rate of return)

$E(I_K) - E(R_Z)$: Risk premium relative to factor k

Moreover, Roll realized that a portfolio that has been adequately diversified possesses a high correlation when it is drawn from a similar asset class and less correlation when diversification occurs across multiple asset groups.

Example: Calculating Expected Return under APT

The following data exists for asset A:

- Risk-free rate = 3%,
- GDP factor beta = 0.40,
- Consumer sentiment factor beta = 0.20,
- GDP risk premium = 2%,
- Consumer sentiment risk premium = 1%

Calculate the expected return for Asset A using a 2-factor APT model.

$$E(R_A) = 0.03 + 0.4(0.02) + 0.2(0.01) = 0.04 = 4\%$$

Note: Both CAPM and APT describe equilibrium expected returns for assets. CAPM can be considered a special case of the APT in which there is only one risk factor – the market factor.

Many investors prefer APT to CAPM since APT is an improved version of CAPM. This is because CAPM is a one-factor model (only the market index is used to calculate the expected return of any security). At the same time, the APT is a multifactor model where numerous indices are used to explain the variation of the expected rate of return of any security.

Multifactor Models

A multifactor model is a financial model that **employs multiple factors** in its calculations to explain asset prices. These models introduce uncertainty stemming from **multiple sources**. CAPM, on the other hand, limits risk to one source – covariance with the market portfolio. Multifactor models can be used to calculate the required rate of return for portfolios as well as individual stocks.

CAPM uses just one factor to determine the required return – the **market factor**. However, the market factor can be **split** up even further into different macroeconomic factors. These may include inflation, interest rates, business cycle uncertainty, etc.

A **factor** can be defined as a variable that explains the expected return of an asset.

A **factor-beta** is a measure of the sensitivity of a given asset to a specific factor. The bigger the factor, the more sensitive the asset is to that factor.

A multifactor appears as follows:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{ik}F_k + e_i$$

Where:

R_i=rate of return on stock i

$E(R_i)$=expected return on stock i

β_{ik}=sensitivity of the stock's return to a one-unit change in factor k

F_k=Macroeconomic factor k

e_i=the firm-specific return/portion of the stock's return unexplained by macro factors

The expected value of the firm-specific return is **always zero**.

The Expected Return of An Asset Using the Single-factor Model

The single-factor model assumes there's just one macroeconomic factor, and appears as follows:

$$R_i = E(R_i) + \beta_i F + e_i$$

$E(R_i)$ is the expected return on stock i. In case the macroeconomic factor has a value of zero in any particular period, then the return on the security will equal its initially expected return $E(R_i)$ plus the effects of firm-specific events.

Example of a Single-factor Model

Assume the common stock of Blue Ray Limited (BRL) is examined with a single-factor model, using unexpected percent changes in GDP as the single factor. Assume the following data is provided:

- Expected return for BRL = 10%
- GDP factor-beta = 1.50
- Expected GDP growth = 4%

Compute the required rate of return on BRL stock, assuming there is no new information regarding firm-specific events.

Solution

We know that:

$$R_i = E(R_i) + \beta_i F + e_i$$

$$= 10\% + 1.5 \times 4\%$$

$$= 16\%$$

The Expected Return of an Asset Using the Multi-factor Model

Example of a Multi-factor Model

Assume the common stock of BRL is examined using a multifactor model, based on two factors:

unexpected percent change in GDP and unexpected percent change in interest rates. Assume the following data is provided:

- Expected return for BRL = 10%
- GDP factor beta = 1.50
- Interest rate factor beta = 2.0
- Expected growth in GDP = 2%
- Expected growth in interest rates = 1%

Compute the required rate of return on BRL stock, assuming there is no new information regarding firm-specific events.

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2$$

$$= 10\% + 1.5 \times 2\% + 2.0 \times 1\%$$

$$= 15\%$$

Hedging Exposures to Multiple Factors

The specific risks (idiosyncratic risks) can be removed by diversification, but the factor betas (systematic risk) can only be removed by hedging strategy. Each factor can be regarded as fundamental security and can, therefore, be utilized to hedge the same factor relative to given security.

Consider an investor who manages a portfolio with the following factor betas:

- GDP beta = 0.4
- Consumer sentiment beta = 0.20

Case 1:

Assume the investor wishes to hedge away GDP factor risk, yet maintain the 0.20 exposure to consumer sentiment. How would they achieve this?

The investor should combine the original portfolio with a 40% short position in the GDP factor portfolio. The GDP factor-beta on the 40% short position in the GDP factor portfolio equals -0.40, which perfectly offsets the 0.40 GDP factor-beta on the original portfolio.

Case 2:

Assume the investor might want to hedge away consumer sentiment (CS) factor risk, yet maintain the 0.40 exposure to GDP. How would they achieve this?

The investor should combine the original portfolio with a 20% short position in the consumer sentiment factor portfolio. The CS factor-beta on the 20% short position in the GDP factor portfolio equals -0.20, which perfectly offsets the 0.20 GDP factor-beta on the original portfolio.

Case 3:

Assume the investor wants to hedge away both factor risks. How would they achieve this?

The investor would have to form a portfolio that's 40% invested in the GDP factor portfolio, 20% in the CS factor portfolio, and 40% in the risk-free asset (note that total = 100%). Let us refer to this portfolio as portfolio H.

Portfolio H can be used to hedge away all the risk factors of the original portfolio. That would involve combining the original portfolio with a short position in portfolio H. The original portfolio betas (0.4 and 0.2) would be perfectly offset by the short position in portfolio H, the hedge portfolio.

The Fama-French Three-Factor Model

One widely used multifactor model that has been developed in recent times is the Fama and

French three-factor model. A major weakness of the APT model is that it is silent on the issue of the relevant risk factors for use. The FF three-factor model puts three factors forward:

- Size of firms
- Book-to-market values
- Excess return on the market

The firm size factor, also known as SMB (small minus big) is equal to the difference in returns between portfolios of small and big firms ($R_s - R_b$).

The book-to-market value factor, also known as HML (high minus low) is equal to the difference in returns between portfolios of high and low book-to-market firms ($R_H - R_L$).

Note: book-to-market value is book value per share divided by the stock price.

Fama and French put forth the argument that returns are higher on small versus big firms as well as on high versus low book-to-market firms. This argument has indeed been validated through historical analysis. Fama and French contend that small firms are inherently riskier than big firms, and high book-to-market firms are inherently riskier than low book-to-market firms.

The equation for the Fama-French three-factor model is:

$$E(R_P) - r = \beta_{PM}[E(R_M) - r] + \beta_{P,SMB}E(SMB) + \beta_{P,HML}E(HML)$$

Where,

$E(R_P)$: is the expected return on portfolio P

r: risk-free interest rate;

$E(R_M - r), E(SMB)$ and $E(HML)$: expected premiums;

$\beta_{PM}, \beta_{P,SMB}, b_{P,HML}$: the coefficients for the time-series regression:

$$R_P - r = a_P + \beta_{PM}(R_M - r) + \beta_{P,SMB}SMB + \beta_{P,HML}HML + \epsilon_P$$

The intercept term, α_p, equals the abnormal performance of the asset after controlling for its exposure to the market, firm size, and book-to-market factors. As long as the market is in equilibrium, the intercept should be equal to zero, assuming the three factors adequately capture all systematic risks. ϵ_i represents random error.

Exam tip: SMB is a hedging strategy – long small firms, short big firms. HML is also a hedging strategy – long high book-to-market firms, short, low book-to-market firms.

Fama and French expanded their model in 2015 by proposing two factors:

- Robust Minus Weak (RMW). RMW is the difference between the return of firms with high (robust) and weak (low) operating profitability.
- Conservative Minus Aggressive (CMA): the difference between the returns of the firms that conservatively invest and those with aggressive kind of investment.

Example: Calculating the Expected Return of a Portfolio Based on the Fama-French Three-Factor Model

A Firm's financial analyst believes the Fama-French dependencies are given in the table below.

	Value
Beta	0.3
SMB	1.25
HML	-0.7

Solution The firm earns an extra 4% yearly due to competitive advantage. Moreover, the firm earns a 15% return on equities, an SMB of 2.5%, and HML of 0% and a risk-free rate of 2%. What is the expected return of the firm?

According to the Fama-French Three-Factor Model the expected return is given by:

$$R_P - r = a_P + \beta_{PM}(R_M - r) + \beta_{P,SMB}SMB + \beta_{P,HML}HML$$

$$R_P - 2\% = 4\% + 0.30(15\% - 2\%) + 2.5\% \times 1.25 - 0.70 \times 0\% = 13.03\%$$

Question

Suzy Ye is a junior equity research analyst at a research firm based in South Korea. For the first time, she is using the multifactor model to compute the return of Wong Kong Corp (WK). She has compiled the following data for the computation of the return:

- Wong Kong's expected return: 7%
- Expected GDP growth: 4.5%
- Expected Inflation: 2.5%
- GDP factor-beta: 1.5
- Inflation factor-beta: 2
- Risk-free rate: 2%

Suppose the actual GDP growth and actual inflation of South Korea are 3% and 2.9%, respectively, then which of the following is an accurate estimate of the return?

A. 7.55%

B. 10.05%

C. 5.55%

D. 18.75%

The correct answer is **C**.

A multifactor model (2-factor model in the given question) only includes the expected return of the stock, macroeconomic factor and the factor-beta, and firm-specific risk, which in this case is zero.

$R_{WK} = E(R_{WK}) + \beta_{GDP}F_{GDP} + \beta_I F_I$

$$= 0.07 + 1.5(0.03 - 0.045) + 2(0.029 - 0.025)$$

$$= 0.07 - 0.0225 + 0.008 = 5.55\%$$